Distributed Power Allocation For OFDM Wireless Ad-Hoc Networks Based On Average Consensus

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***Abstract*— This paper addresses the problem of optimal power allocation in multiuser OFDM ad-hoc networks. Our objective is to maximize the weighted rate-sum of users under total transmit power constraint. Maximizing a weighted rate-sum is more amenable whereas appropriate weight assignment to different users, guarantees fairness among users. We propose a distributed algorithm for optimal power allocation based on reaching a consensus among users for power allocation. The proposed algorithm is tractable in the sense of computational complexity and requires only local information at each node. Simulation results and the analysis of the algorithm are presented to support the proposed idea.**

***Index Terms*-OFDM, Power Allocation, Ad-Hoc Network, Waterfilling, Consensus Algorithms.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is an attractive solution for future wireless communication networks. OFDM can provide a high performance physical layer and medium access control thanks to its ability to combat ISI and multipath fading. Additionally, through dynamic subcarrier allocation, OFDM can exploit multiuser diversity which is inherent in the multiuser wireless networks. The most challenging issue in OFDM systems is the problem of subcarrier and power allocation in a multiuser network in order to minimize the total transmit power or maximize the total data rate or a utility function of data rate of users. The problem of optimal subcarrier and power allocation for a multiuser OFDM network has attracted many research interests and are investigated by many researchers ([1]- [8]). In [1], optimal subcarrier and power allocation has been carried out to minimize the total transmit power of all users. In [2], the authors outlined the problem of data rate maximization while achieving proportional fairness among users. High computational complexity makes the optimal subcarrier and power allocation impractical. In order to reduce complexity, several sub-optimal subcarrier and power allocation algorithms have been presented. [3] proposes a sub-optimal algorithm for subcarrier assignment with uniform power allocation. In [5] the authors outlined a joint suboptimal subcarrier and power allocation for multiuser OFDM. The [6] [7] have addressed the problem of optimal power and subcarrier allocation while maximizing a utility of data rate of users.

All of the aforementioned studies have focused on a centralized schemes and consequently, are not practical for distributed implementation which is applicable for wireless ad-hoc networks. To the best of our knowledge, only little studies such as [8] have considered multiuser OFDM in ad-hoc networks. In [8] the authors addressed the problem of power minimization in an ad-hoc network while maintaining a fixed data rate on each link.

In this paper, we consider fixed subcarrier allocation and address the problem of optimal power allocation to maximize the weighted rate-sum of nodes with total transmit power constraint. Maximizing a weighted rate-sum with approperiate weights can guarantee fairness among users, which can not necessarily be achieved with uniform weights. We derive optimal power allocation for subcarriers and propose an algorithm to perform optimal power allocation in a distributed way. Our algorithm is based on consensus algorithms in which all nodes try to reach a consensus or agreement on a desired unknown parameter in a distributed fashion ([9]- [11]).

This paper is organized as follows: in section II we present the system model and formulate our problem. In section III we derive optimal power allocation for each subcarrier and in section IV we propose a distributed power allocation algorithm. Numerical resluts are presented in section V and section VI involves conclusion and future work issues.

II. System Model

We consider an OFDMA-based wireless ad-hoc network. We model the topology of the network by an undirected graph $G = (E, V)$ with vertex set $V = \{1, 2, \ldots, K\}$ and edge set $E = \{(i, j)|i, j \in V\}$ denoting the set of nodes and links, respectively. For edge set, we have $(i, j) \in E$ if and only if there is a connection between node i and j. We denote the neighbors of node i by $N_i = \{j \in V|(i, j) \in E\}$. In this paper, we focus on the fixed subcarrier assignment scheme, i.e. each node according to its transmit data rate, has a fixed predetermined number of subcarriers. We also denote the set of subcarriers assigned to node k by S_k, whose cardinality, $|S_k|$, represents the number of subcarriers of node k. As we consider an OFDMA network with no clustering, the subcarrier sets of all links are disjoint sets. Transmit power, channel coefficient and noise power on the nth subcarrier of the node k are represented by $P_{k,n}$, $h_{k,n}$ and $n_{k,n}$, respectively. Therefore, the maximum data rate that can be sent on the nth subcarrier is given by [12] and [13]:

$$c_{k,n} = \log(1 + \frac{P_{k,n}h_{k,n}^2}{\Gamma n_{k,n}}) \quad (1)$$

where Γ is the SNR-gap. The SNR-gap defines the gap between a practical coding and modulation scheme and the channel capacity and depends on the coding and modulation scheme used for a specific probability of error. The total data rate of node k is given by:

$$R_k = \sum_{n \in S_k} c_{k,n} \tag{2}$$

Although (2) seems to involve only single-hop transmission schemes, in the aforementioned model we have also modeled multi-hop ones. In fact, for a known routing policy, each node has a specific amount of data to be sent to its neighbor(s) which consists of its own data and the others' data to be relayed through it. In this respect, such a model would involve multi-hop transmission scheme.

Our objective is to maximize the weighted rate-sum of all nodes under the constraint that total power of all users can not exceed a maximum value. As we consider fixed subcarrier assignment, our goal is to find the optimal power allocation of subcarriers. The optimization problem can be formulated as:

$$\max_{P_{k,n}} \sum_{k=1}^{K} \alpha_k R_k \tag{3}$$

subject to:

$$\sum_{k=1}^{K} \sum_{n \in S_k} P_{k,n} \leq P_{\max} \tag{4}$$

$$S_1, S_2, \ldots, S_K \text{are all disjoint} \tag{5}$$

$$S_1 \cup S_2 \cup \ldots \cup S_K = \{1, 2, \ldots, N\} \tag{6}$$

where $P_{\max}$ is the upper bound of the total transmission power, α_k is the weight assigned for node k according to the priority of it with regard to others and N denotes the number of subcarriers in the system. For notational convenience in the final solution, α_k's are normalized to satsify the following condition:

$$\frac{1}{K} \sum_{k=1}^{K} \alpha_k = 1 \tag{7}$$

Appropriate weight assignment can achieve fairness across users and prevents allocating more resources to nodes with good channels. In order to make the problem formulation more tractable, we introduce subcarrier sharing factors, $\rho_{k,n}$. If subcarrier n is assigned to node k, $\rho_{k,n} = 1$, otherwise $\rho_{k,n} = 0$. For notational convenience we define:

$$g_{k,n} = \frac{h_{k,n}^2}{\Gamma n_{k,n}} \tag{8}$$

III. Optimal Power Allocation

In this section, we derive optimal power allocation for all subcarriers and present a theorem which proves that such power allocation can be done distributedly, with arbitrarily small error. Using the standard optimization methods [14], the Lagrangian can be written as:

$$\mathcal{L} = \sum_{k=1}^{K} \sum_{n=1}^{N} \alpha_k \rho_{k,n} \log(1 + P_{k,n} g_{k,n}) - \lambda \sum_{k=1}^{K} \sum_{n=1}^{N} P_{k,n} \tag{9}$$

where λ represents the Lagrange multiplier for constraint (4). By taking the derivative of $\mathcal{L}$ with respect to $P_{k,n}$, we obtain the necessary condition for the optimal solution:

$$\frac{\partial \mathcal{L}}{\partial P_{k,n}} |_{P_{k,n}^*} = \frac{\alpha_k \rho_{k,n} g_{k,n}}{1 + P_{k,n} g_{k,n}} - \lambda = 0 \tag{10}$$

$$P_{k,n}^* = \rho_{k,n} [\frac{\alpha_k}{\lambda} - \frac{1}{g_{k,n}}]^+ \tag{11}$$

The optimal power allocation obtained above, is somewhat similar to classical waterfilling, but different weights for different nodes have led to different water levels, i.e. $\frac{\alpha_k}{\lambda}$. Indeed, this is multi-level waterfilling, which has been proposed recently in the context of utility-based resource allocation [6]. An illustrative example of multi-level waterfilling is depicted in Fig. 1. As we would like to maximize a weighted rate-sum of users, this is equivalent to fill water to each node's *bowl* whose depth is proportional to the weight assigned to it.

In a centralized scheme, a center calculates λ such that constraint (4) will be satisfied and then sends this value to each user. In a distributed scheme, such as ad-hoc networks, the value of λ must be calculated distributedly. In the next section, we propose an algorithm to calculate λ distributedly based on distributed averaging. Before we proceed to our algorithm, we state how λ is related to the average of nodes' data.

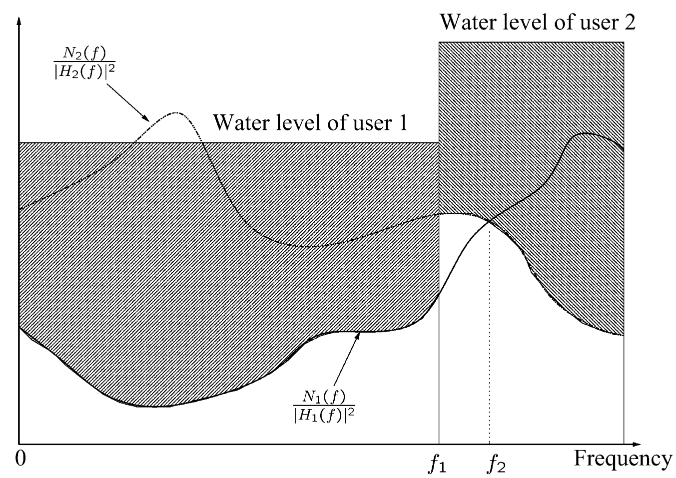

Fig. 1. Multi-level Waterfilling

Theorem 1: In the waterfilling process, the Lagrange multiplier, λ, which determines the water level is given by:

$$\lambda = \frac{1}{\bar{L} + \tilde{L}} \tag{12}$$

where $\bar{L}$ is the average of water level of subcarriers for uniform power allocation and $\tilde{L}$ is the sum of negative powers

- allocated using $\bar{L}$ - averaged over strong subcarriers, i.e. subcarriers with non-negative power.

Proof: Water level after a waterfilling process, is given by [13]:

$$\frac{1}{\lambda} = \frac{1}{|D|}(P_{\max} + \sum_{n\in D} \frac{1}{g_n}) \tag{13}$$

where D is the set of subcarriers with non-negative power, and $\bar{D}$ is the set of ones with zero power. Clearly, $D \cup \bar{D} = \{1, 2, \ldots, N\}$. Allocating uniform power to each subcarrier, yields

$$L_n = \frac{P_{\max}}{N} + \frac{1}{g_n} \tag{14}$$

where L_n denotes the initial water level of each subcarrier. We denote the average of L_n's by $\bar{L}$. Therefore we have

$$\bar{L} = \frac{1}{N}\sum_{n=1}^{N}(\frac{P_{\max}}{N} + \frac{1}{g_n}) \tag{15}$$

Power allocation according to the average water level, $\bar{L}$, is given by

$$P_{k,n} = \bar{L} - \frac{1}{g_n} \tag{16}$$

Negative powers in 16 introduce error to the final water level. In order to overcome to this problem, we should share the negative powers among strong subcarriers. In this respect, we average the negative powers over strong subcarriers to be added to the average water level. The avarege of negative power levels over strong subcarrier is given by:

$$\tilde{L} = \frac{1}{|D|}\sum_{n\in\bar{D}} P_{k,n} \tag{17}$$

Adding the $\tilde{L}$ to average water level of subcarriers, yields the final water level, L_{final}, as

$$\begin{aligned}\bar{L} + \tilde{L} &= \bar{L} + \frac{1}{N - |\bar{D}|}\sum_{n\in\bar{D}} P_{k,n} \qquad (18)\\ &= \bar{L} + \frac{1}{N - |\bar{D}|}\sum_{n\in\bar{D}}(\bar{L} - \frac{1}{g_n})\\ &= \bar{L}(1 + \frac{|\bar{D}|}{N - |\bar{D}|}) - \frac{1}{N - |\bar{D}|}\sum_{n\in\bar{D}}\frac{1}{g_n}\\ &= \frac{N}{N - |\bar{D}|}\bar{L} - \frac{1}{N - |\bar{D}|}\sum_{n\in\bar{D}}\frac{1}{g_n}\end{aligned}$$

combining (15) and (18) yield:

$$\begin{aligned}\bar{L} + \tilde{L} &= \frac{1}{N - |\bar{D}|}(\frac{P_{\max}}{N} + \sum_{n=1}^{N}\frac{1}{g_n}) - \frac{1}{N - |\bar{D}|}\sum_{n\in\bar{D}}\frac{1}{g_n} \qquad (19)\\ &= \frac{1}{N - |\bar{D}|}(\frac{P_{\max}}{N} + \sum_{n\in D}\frac{1}{g_n})\\ &= \frac{1}{\lambda}\end{aligned}$$

Therefore

$$\lambda = \frac{1}{\bar{L} + \tilde{L}} \tag{20}$$

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IV. DISTRIBUTED POWER ALLOCATION ALGORITHM

As discussed above, optimal power allocation in an ad-hoc network necessitates performing waterfilling in a distributed manner. In order to perform multi-level waterfilling distributedly, each node should have knowledge about its own water level, i.e. $\frac{\alpha_k}{\lambda}$. Intuitively, we may think of multi-level waterfilling as the classical one, by scaling noise and power of user k with $\frac{1}{\alpha_k}$. The larger the weight of user k, the more would be the power, and the network has more incentive to allocate power to it. In other words, nodes with large weights have permission to announce their SNR scaled by α_k in order to absorb more water from available resources. Based on the above heuristic, all nodes only need to have agreement on the value of $\frac{1}{\lambda}$. Toward this, we propose a distributed algorithm to reach a consensus on the value of $\frac{1}{\lambda}$. Our algorithm is based on distributed averaging called *consensus averaging* which calculates the average of a group of nodes in a distributed fashion by reaching a consensus among them.

The problem of distributed averaging has been extensively studied recently in the context of data fusion over networks. So far several ways for distributed averaging have been proposed [9], [15], [16]. In this paper, we adopt the method proposed by Xiao *et al.* [15] which involves single-hop transmissions and requires local information of the network. In the method presented in [15], each node exchanges its data with its neighbors and updates its data according to a weighted sum of its data and that of its neighbors, iteratively. The update equation can be written as [15]

$$x_i(t+1) = W_{ii}x_i(t) + \sum_{j\in N_i} W_{ij}x_j(t) \tag{21}$$

where $x_i(t)$ represents data of node i after t iterations and W_{ij}'s denotes the weights of user i. Aforementioned updating in each node continues until convergence is achieved. It has been proved [15] that with appropriate choice of weights and after large enough iterations, all nodes' data will converge to the average of the entire network, even though nodes only have local information about the network. In this respect, nodes can obtain knowledge about the value of a parameter which requires complete information of all nodes, such as water level in the waterfilling.

It should be noted that there are several choices of weights ([9], [15]). One famous and simple one is Metropolis which requires only local information and converges very fast compared to other ones. Metropolis weight matrix is defined as:

$$W_{ij} = \begin{cases} \frac{1}{1+\max\{d_i, d_j\}} & \{i,j\} \in N_i \\ 1 - \sum_{i,k\in N_i} W_{ik} & i = j \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

The weights in (22) is applicable for averaging with uniform weighting. Therefore, usage of (22) is restricted to uniform subcarrier assignment schemes. In order to evaluate a weighted average of nodes' data, which is useful for non-uniform subcarrier assignment schemes, the update equation 23 should be modified as the following

$$x_i(t+1) = W_{ii}\beta_i x_i(t) + \sum_{j \in N_i} W_{ij}\beta_j x_j(t) \tag{23}$$

where β_j denotes the sharing weight of x_j in the averaging. For simplicity, we embed the β_j factor in the initial values of data. For a non-uniform subcarrier assignment scheme, the proposed algorithm initializes with an initial water level given by

$$L_{1k} = \beta_k\Big(\frac{P_{max}|S_k|}{N} + \sum_{n \in S_k} \frac{1}{g_{k,n}}\Big) \tag{24}$$

The proposed algorithm performs distributed multi-level waterfilling. The algorithm, motivated from Theorem 1, has two major loops. The first loop is devoted to evaluate the average water level of all subcarriers iteratively, based on the consensus averaging with modified Metropolis weights. As mentioned above, the average water level of all subcarriers is equivalent to a weighted average of nodes' water. At the end of first loop, each node knows the average water level of all subcarriers. In the next step, each node performs a waterfilling according to the derived average level of water. In high SNR regime over entire bandwidth, all subcarriers will be waterfilled with a positive value of water (i.e. power), but if some of subcarriers have low SNR values, they may be waterfilled with negative power and thus they can introduce errors to the exact water level. The error introduced is more critical when more subcarriers suffer from such low SNR values. In order to correct the error inroduced in the average water level, each node measures the negative power (water) it has already assigned to its weak subcarriers averaged over other subcarriers. In the next loop, all nodes try to find the average of the error, iteratively. Finally, according to Theorem 1, each node calculates the final water level by adding the average error to the average water level measured in the first loop and then scale the result according to its weight in the maximization. The algorithm terminates with power allocation according to the final water level. The proposed distributed optimal power allocation is shown below as algorithm 1.

Algorithm 1 Consensus Power Allocation

Initialization

$x_{1k}(0) = \beta_k(\frac{P_{max}|S_k|}{N} + \sum_{n \in S_k} \frac{1}{g_{k,n}})$

Step 1

While $t <$ `max_iteration` AND `status`$\neq$`converged`

$x_{1k}(t+1) = W_{kk}x_{1k}(t) + \sum_{j \in N_k} W_{kj}x_{1j}(t)$

end while

Step 2

$x_{2k} = \frac{\beta_k}{|S_k| - |\bar{S_k}|} \sum_{n \in \bar{S_k}} P_{k,n}$

While $t <$ `max_iteration` AND `status`$\neq$`converged`

$x_{2k}(t+1) = W_{kk}x_{2k}(t) + \sum_{j \in N_k} W_{kj}x_{2j}(t)$

end while

Step 3

$L_k = \alpha_k(x_{1k} + x_{2k})$

for $n \in S_k$

$P^{\star}_{k,n} = [L_k - \frac{1}{g_{k,n}}]^+$

end for

end

Algorithm 1. Consensus Power Allocation

V. Simulation Results and Analysis

In this section, performance of the proposed algorithm is analyzed through simulation and is compared with the centralized scheme. We have considered a network of 16 randomly placed nodes over $[0, 100] \times [0, 100]$ field with a connectivity degree 0.35, which defines the range in which nodes can see each other. In other words, nodes with distance not more than 0.35, are assumed connected. Each node wants to transmit or relay data to its destination(s) according to a routing policy. We also assume that total system bandwidth is 10MHz and there are 64 subcarriers in the OFDM system. The noise power spectral density at each subcarrier is $10^{-14}W/$Hz and the SNR gap is supposed to be 8.8dB. Assuming fixed and uniform subcarrier assignment, each node has to send its data over a set of 4 predetermined subcarriers.

$$h_{ij} = \frac{G_{ij}\alpha_{ij}}{d_{ij}^4} \tag{25}$$

We consider an environment with path loss and rayleigh fading and adopt the channel model presented in [8]. In this respect, the channel coefficient from node i to node j is modeled as where d_{ij}^{-4} and α_{ij} represent the path loss and rayleigh fading parameters, respectively. Transmitter and receiver antenna gains are combined in the G_{ij} factor. In order to maximize the overall data rate while trying to allocate equal rate to all nodes, α_k's are assumed to be 1. Sum of the user's data rate with the proposed power allocation and uniform power allocation for a wide range of values of $P_{\max}$ is depicted in Fig. 2. Performance improvement which is defined as the ratio of increase in the overall rate and total rate is also shown in Fig. 3. It is worth noting that as total transmit power, $P_{\max}$,

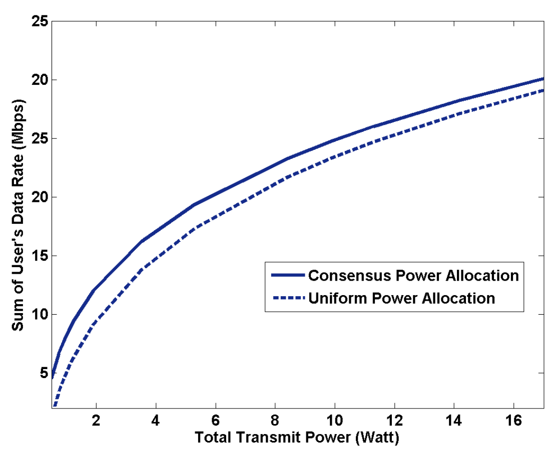

Fig. 2. Sum of User's Data Rate vs. Total Transmit Power

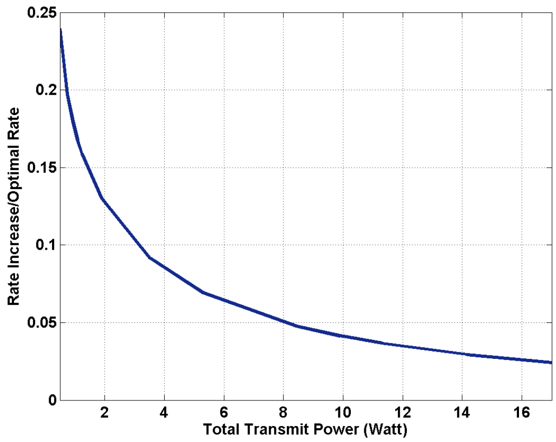

Fig. 3. Proportional Rate Increase vs. Total Transmit Power

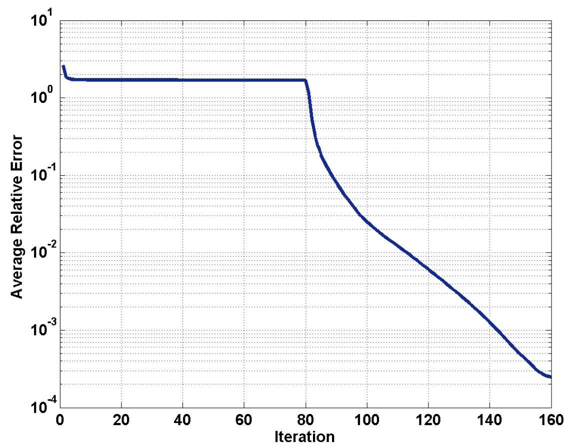

Fig. 4. Convergence Behaviour

increases, the achieved gain diminishes and uniform power allocation becomes optimal. Fig. 4 shows the convergence behavior of the proposed algorithm averaged over all nodes. As shown in Fig. 4, it is clear that the proposed algorithm converges very fast thanks to good convergence behaviour of Metropolis weights. When small errors are acceptable, the algorithm converges with a few iterations and therefore its overhead becomes negligible. In this respect, it is clear from Fig. 4 that even with small iterations, centralized scheme achieves slightly better performance with regard to distributed scheme and hence performance degradation of distributed algorithm is negligible.

VI. Conclusion and Future Work

In this paper, we introduced a distributed power allocation algorithm for OFDM wireless ad-hoc networks with fixed subcarrier assignment in order to maximize a weighted rate-sum of nodes. The proposed algorithm is based on reaching a consensus among nodes for the *level of water* to perform a multi-level waterfilling in each node. The proposed algorithm converges fast and is tractable from computational complexity point of view. Simulation results confirm that the performance degrdation introduced by distributed power allocation is quite small with regard to centralized scheme. In order to increase the convergence rate of the proposed algorithm, adaptive weights can be used as proposed in [17], for consensus averaging in the algorithm instead of Metropolis. As an extention to current paper we are considering joint power allocation and subcarrier assignment in ad-hoc OFDM network, distributedly.

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