Compensating for the Latency of Data Acquisition for Localization in Mobile Wireless Sensor Networks

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Abstract—The problem of latency in data acquisition, which is specific to the case of mobile wireless sensor networks, has been introduced and investigated. In this regard, the properties of the so-called Data Acquisition Latency Error, and the way it affects the performance of the respective localization schemes in a mobile network were discussed through both theoretical analysis and simulations. Moreover, a weighted version of lateration scheme was introduced and shown to outperform the traditional one, while tracking a mobile node. This work is to the best of our knowledge the first to have addressed the aforementioned problem in mobile wireless sensor networks, and to have offered a solution for its alleviation.

I. INTRODUCTION

Despite the extensive body of research on localization in wireless sensor networks (WSN), the amount of research that has been devoted to the study of localization in mobile WSN, is slim. The majority of the existing literature have been focusing on localization in static sensor networks, where localization is basically a one time or low-frequency activity. In a static case the sensor nodes are either essentially constant or otherwise move in a manner that can be tolerated by the localization method, i.e. are much slower compared to the time taken by the localization and its frequency.

In the mobile case, on the other hand, due to the constant changes in the position of the nodes, the localization process should be repeatedly invoked. The authors in [1], have suggested a set of protocols which will monitor and adjust the frequency with which localization is performed. Reference [2] considers the problem of designing the optimum length of the averaging window for RSSI measurements in a mobile network, where fading and motion account for two opposite behaviors from an averaging point of view, giving rise to an interesting trade-off. To be more specific, the longer window widths, although will help mitigate the fading effects, would be translated into a larger change in the position of the node, as the node is mobile. Finally, the Monte-Carlo Localization (MCL) technique in [3] leverages the changes in network connectivity, caused by the node’s movements, to update the samples of probability distribution function of the position of a target node. Reference [4] introduces an enhanced version of MCL, called MSL, which can work with both mobile and static networks.

None of the above however, have addressed the time-shift between data acquisition and localization, which is inherent in mobile WSN and can cause the tracking scheme to fail, no matter how precise the distance data are, at the point of measurement. In other words, as a node moves across the field, the range, angle, or connectivity data measured by the node at some point in time may no longer be valid at a later time, when it is used to derive the node’s unknown position.

The rest of this paper is organized as follows. This introduction is followed by Section (II), which defines the Data Acquisition Latency (DAL) error and presents a theoretical analysis of its properties. Next in Section (III) we show how the distance data are used to estimate the unknown coordinates of a target node using a traditional lateration scheme, which we will then improve in section (IV) to help alleviate the effects of data acquisition latency in the localization for mobile WSN. The simulations in Section (V) supplement the discussions of the previous three sections, and Section (VI) concludes this paper.

II. THE LATENCY IN DATA ACQUISITION FOR MOBILE WIRELESS SENSOR NETWORKS

In the localization for mobile wireless sensor nodes, the inherent latency of data acquisition and data shift caused by the difference between the time of data measurement and the time of localization, can seriously disrupt the operation of the existing localization methods even if the range or distance data are precisely known. Figure (1) demonstrates how the so-called Data Acquisition Latency Error can degrade the performance of a traditional lateration-based localization scheme, even if the measured distance data are error free at each instance of measurement.

In this paper, the distance measurements are assumed to be between a single mobile target node, whose location is the desired unknown, and N arbitrarily located anchor nodes with known and constant positions. The measured distances are then used in a lateration-based framework to determine the location of the target node, as described in section (III).
Figure (2) exhibits how the distance data measured at some point of time is used to determine the node’s position at a later point. This phenomenon, which we refer to as Latency in Data Acquisition can be the source of DAL error that is the focus of this section.

Moreover, the maximum error in the measured distance from the k-th anchor node, \(d_k\), caused by the Data Acquisition Latency can be formulated as follows (assuming that the magnitude of node’s maximum speed \(V_{\text{max}}\) is known):

\[
|d_k(T_i^N) - d_k(T_i^k)|_{\text{max}} = |V_{\text{max}}|(T_i^N - T_i^k) = |V_{\text{max}}|(N - k)t
\]

This corresponds to the case when throughout the rest of the localization period (from \(T_i^k\) to \(T_i^N\)), the target node moves in the direction of the line connecting its position at \(T_i^k\) to the the k-th anchor node, and at a speed equal to \(V = |V_{\text{max}}|\).

In order to find the maximum overall error in one Beaconsing Period, we set \(k\) in (1) to 1, which results in,

Maximum Distance Error = \(|V_{\text{max}}|(N - 1)t\) (2)

Equation (2) offers a valuable insight into the nature of DAL error and the factors affecting it. In particular it is clear that:

- The error increases with the increasing number of Anchor Nodes (\(N\)).
- Increasing the sampling time (\(t\)) will degrade the operation of the localization scheme in a mobile WSN.
- Changing the Beaconsing Period (\(T\)) does not affect the Data Acquisition Latency error.
- The increase in the mobility of the target nodes (increasing \(|V_{\text{max}}|\)) exacerbates the error.
- Last but not least, according to the relation derived in (1) the value of the error is the largest at the first time-slot \((k = 1)\) and decreases to zero as we approach the end of a Beaconsing Period \((k = N)\).

The above effects are simulated in section (V-B). The relationship between localization error and distance measurement error is explored in the next section.

III. RELATION BETWEEN LOCALIZATION ERROR AND DISTANCE MEASUREMENT ERROR

Suppose that the following set of inter-nodal distances have been measured:

\[
(d_{11}, d_{21}, \ldots, d_{NN})^T
\]

where, \(d_{ii}, i = 1, \ldots, N\), represents the distance between the target node and the i-th anchor, at the time of its measurement. To emphasize the error in distance measurements we may use ‘~’ marks. Each of the measured distances of (3) in turn, corresponds to a nonlinear equation in terms of the unknown coordinates of the target node, \((X_T, Y_T)\), and of the i-th anchor node, \((X_{A_i}, Y_{A_i})\), which are known. This equation in the 2D case will take the form:

\[
(X_T - X_{A_i})^2 + (Y_T - Y_{A_i})^2 = d_{ii}^2, \ i = 1, \ldots, N
\]

thus resulting in a set of \(N\) nonlinear (quadratic) equations. It can be shown that the set of \(N\) nonlinear equations given by (4) can be transformed into the following linear system
of \( N - 1 \) equations in terms of the target node’s unknown coordinates \((X_T, Y_T)\) [5]:

\[
\begin{pmatrix}
2(X_{A_N} - X_A) & 2(Y_{A_N} - Y_A) \\
\vdots & \vdots \\
2(X_{A_N} - X_{A_{N-1}}) & 2(Y_{A_N} - Y_{A_{N-1}})
\end{pmatrix}
\begin{pmatrix}
X_T \\
Y_T
\end{pmatrix} = \begin{pmatrix}
d_1^2 - d_2^2 \\
\vdots \\
(\hat{d}_{N-1}^2 - d_N^2)
\end{pmatrix} + \begin{pmatrix}
d_1^2 - d_N^2 \\
\vdots \\
(\hat{d}_{N-1}^2 - d_N^2)
\end{pmatrix}
\]

(5)

Denoting the left-hand-side coefficients matrix in (5) by \( A \), the unknown coordinates by \( x \), and the right-hand-side by \( b \), (5) can be rewritten as:

\[
A \times x = b
\]

(6)

where for \( N = 3 \) non-collinear anchors, \( A \) is a non-singular \( 2 \times 2 \) matrix. However, so as to reduce the influence of errors in the observations (distance measurement errors), one would like to use a greater number of measurements than the number of unknown parameters in the model [6]. The resulting problem is to solve an overdetermined linear system of equations. The optimum solution, \( \hat{x} \), should minimize the Euclidean error vector (residual vector) norm, \( ||Ax - b||^2 \), and is given by:

\[
A^T Ax = A^T b
\]

(7)

which upon solving for \( \hat{x} \) yields:

\[
\hat{x} = (A^T A)^{-1} A^T b
\]

(8)

The relations (8) and (5) can be utilized to show how the distance measurement errors are mapped into the localization error, through the lateration procedure:

\[
e^2 = \begin{pmatrix}
\hat{d}_1^2 - d_1^2 + d_N^2 - d_N^2 \\
\vdots \\
\hat{d}_{N-1}^2 - d_{N-1}^2 + d_N^2 - d_N^2
\end{pmatrix}^T
\]

(9)

\[
\cdots (B^{-1})^T B^{-1} \begin{pmatrix}
\hat{d}_1^2 - d_1^2 + d_N^2 - d_N^2 \\
\vdots \\
\hat{d}_{N-1}^2 - d_{N-1}^2 + d_N^2 - d_N^2
\end{pmatrix}
\]

(10)

where \( B = (A^T A)^{-1} A^T \).

1Vectors are expressed by bold face lower case letters, superscript \( ^T \) denotes the matrix transpose, and \( \varepsilon[.] \) indicates the mathematical expectation.

It should be noted that the localization error, \( e \), has been defined as the Euclidean distance between the detected location and the real position of the target node at the end of a beaconing period \( T \), defined in figure (2).

In the next section, we introduce a modification to the above procedure that will enhance its suitability for a mobile target node, suffering from DAL.

IV. WEIGHTED LEAST SQUARES

In the previous section, the least squares method was utilized to derive the unknown coordinates of the target nodes from the measured distances. Gauss-Markoff theorem ensures that when the errors have zero expectations, are uncorrelated, and have equal variances, the solution, given by (8), will be the best linear unbiased estimator (BLUE) [6].

Now consider the case of the least squares coordinate estimation scheme described in section (III), with the measured distances given as in (3). Assuming that the measured distances suffer from the DAL error, described in section (II), we can write using equation (1):

\[
|\hat{d}_1 - d_1|_{\text{max}} > |\hat{d}_2 - d_2|_{\text{max}} > \cdots > |\hat{d}_{N-1} - d_{N-1}|_{\text{max}} > |\hat{d}_N - d_N|_{\text{max}} = 0
\]

(10)

where “\( \hat{\cdot} \)” denotes the measured values, and it is also assumed that DAL is the only source of error in the measurements. Obviously, (10) implies that the assumption of equal variances cannot be applied to the observations anymore, i.e. the elements of the observation vector, \( b \), in (6) are not known with the same accuracy and the errors do not have equal variances. Hence, the application of equation (7) to estimate the unknown coordinates of the target node will not result in the BLUE of the target node’s position anymore. Aitken(1935), introduced a well-known extension to the Gauss-Markoff Theorem that provides a BLUE, for the case where the covariance matrix of the errors, \( \varepsilon[.] \), is non-scalar and is given by:

\[
\varepsilon[\varepsilon]\hat{\varepsilon} = \sigma^2 W
\]

(11)

The errors should still have zero means. Accordingly, with the error covariance matrix given as in (11), the BLUE of \( \hat{x} \) in (6) would be the solution of \( \min_{\hat{x}} \{ (A\hat{x} - b)^T W^{-1} (A\hat{x} - b) \} \), and is given by the normal equations [6]:

\[
A^T W^{-1} A\hat{x} = A^T W^{-1} b
\]

(12)

Matrix \( W \) in (12) is assumed to be symmetric, positive-definite. For \( W = I \), identity matrix, we get the special case of the linear model already discussed in section (III).

Now, peering into the right-hand-side of equation (5) it will dawn on us that the only part which is prone to the DAL error is the vector:

\[
\hat{d} = (d_1^2, \ldots, d_{N-2}^2)^T
\]

(13)

Accordingly, the covariance matrix for the errors in the observation vector, \( b \), will be given by the covariance matrix of the vector, \( \hat{d} \):
\[
\text{COV}[\mathbf{b}] = \varepsilon[(b_i - \bar{e}_i)(b_j - \bar{e}_j)]
\]
\[
= \varepsilon[(d_i^2 - \bar{\varepsilon}_i^2)(d_j^2 - \bar{\varepsilon}_j^2)]
\]

(14)

Based on the simulation results in section (V), for relatively random movements of the target node it would be fair to assume that the DAL errors in measuring \(d_i\), \(i = 1, \ldots, N-1\), have zero-means, are uncorrelated, and are uniformly distributed between \(\pm|V_{\text{max}}|(N-i)t\), \(i = 1, \ldots, N-1\). Next using functions of random variables [7], the probability distribution function, \(f_{D_i^2}(d_i^2)\), for the elements of vector \(\mathbf{d}\) can be shown to be given by \(\frac{1}{4d_i^2|V_{\text{max}}|(N-i)t}\) for \(d_i^2 \leq |V_{\text{max}}|(N-i)t\) and 0, otherwise. Finally the covariance matrix in (14) can be computed as:

\[
W_{ij} = \begin{cases} 
0 & i \neq j \\
\frac{1}{4}(d_i^2|V_{\text{max}}|(N-i)t)^2 - \frac{(V_{\text{max}}(N-i)t)^4}{15} & i = j 
\end{cases}
\]

(15)

The diagonal matrix \(W = (W_{ij})_{(N-1) \times (N-1)}\) given by (15) can be used in the context of the normal equations in (12) to derive a weighted version of the linear system (7), in which the more recent measurements are regarded with a higher value and given a larger weight, thus alleviating the effects of DAL error. As a concluding remark, it is worth noting that since this weighting has been proposed as a modification within the framework of the traditional lateration scheme and can be calculated in advance for a given network, it does not impose any additional computational burden. The results of simulations in section (V-B) confirm the superiority of the proposed approach in the mobile case.

V. SIMULATION RESULTS AND DISCUSSION

A. Properties of the Data Acquisition Latency Error

In the preceding sections a theoretical analysis of the properties of DAL error, was presented. In this section we have used MATLAB’s simulation environment to further investigate the properties of this error. Mobility models were exploited to simulate the random movement of the target node across the localization field [8]. Figure (3) depicts the original path and the localized positions, which are obtained using the traditional lateration scheme. Figures (4) shows how \(e_1\) tends to take larger magnitudes, while \(e_4\) the smaller ones. Nonetheless, they still have their random fluctuations, which are due to the various directions that the target node takes with respect to different anchor nodes. Sharp autocorrelation peaks for \(e_1\) to \(e_4\) in figure (7), also the approximately zero averages and random distributions of the magnitudes observed in figure (6) suggest that it would be reasonable to assume a random behavior for the errors subjected to the conditions imposed on the maximum error by equation (1). Moreover, the sharp overall autocorrelation peak depicted in figure (5), which is resulted from the autocorrelation of the sequence in figure (4), as well as the relatively low and noisy magnitude of the mutual cross-correlation functions depicted in figure (8), suggest that the error sequences \(e_1\) to \(e_4\) are reasonably uncorrelated. In lack of any further knowledge, and with the apparent distributed-ness of the error magnitudes between the \(\pm|V_{\text{max}}|(N-i)t\) bounds provided by equation (1), we will assume when needed, that the error sequence \(e_i\), \(1 \leq i \leq N-1\), is a random process, uniformly distributed between \(\pm|V_{\text{max}}|(N-i)t\). Finally it should be highlighted that the above results are correct only when the movement of the target node within the localization field is relatively random.

B. Localization Error Due to the Data Acquisition Latency

Equation (2) happens to be an effective tool when it comes to the prediction of the effects of \(t\) and \(N\) on the Distance and localization errors. Figures (9), (10), and (11) depict the effects of localization period (\(T\)), sampling time (\(t\)), and number of anchor nodes (\(N\)), respectively. Except for the one under investigation the rest of the parameters for the following
The Effect of Localization Period on the Error Caused by the Data Acquisition Latency

Average Distance Measurement Error
Average Localization Error Using Traditional Lateration
Average Localization Error Using the Weighted System

Fig. 9. Effect of Beaconing Period (T)

simulations are kept constants at common values. Moreover, the simulations are performed on a common path generated by using mobility models [8], which are exploited to simulate the movement of a single target node across N constantly located anchor nodes in the localization field. As shown in the figures the proposed weighted system outperforms the traditional lateration scheme under all circumstances.

1) Effect of Beaconing Period: According to Figure (9) and equation (2) the average DAL error does not depend on the beaconing period \( T \); but rather the variations that are observed in the graph are due to the random nature of the error. Moreover, the way the localization error plot and the distance error plot follow one another, can be illuminative to equation (9) and the discussions of section (III).

2) Effect of Sampling time: Figure (10) illustrates the effect of Smapling time, \( t \), on the error performance of the localization scheme for a mobile case. As already pointed out, the distance error caused by DAL increases, with increasing sampling time \( t \). Nonetheless, a longer sampling time, corresponding to a longer averaging window, is often necessary to improve the accuracy of equations, relating the average path-
loss to the transmitter-receiver separation [9]. Therefore, the proper choice of sampling time, \((t)\), poses a fundamental trade-off in the design of the localization schemes for mobile WSN. Increasing sampling time \((t)\) on the one hand increases the reliability of the measured distance data by improving their accuracy and correlation with the measured RSSI, and on the other hand increases DAL error, thus degrading the overall localization performance for a mobile case. Since the latter factor becomes less dominant with the decreasing speed and mobility of the sensor nodes, it is reasonable to assume that an optimum solution will take into account the effect of the target node’s speed, hence favoring longer sampling times for lower speeds.

3) Effect of the Number of Anchor Nodes: Figure (11) demonstrates the simulation results for the localization of a mobile target node with different number of anchor nodes \((N)\) put across the localization field. As predicted by equation (2), the localization performance degrades with increasing \((N)\). However, the relation between the number of anchor nodes and the localization error is made complex due to the fact that an increase in the number of anchor nodes will not only increase the absolute value of elements of distance error vector in equation (9) as suggested by equation (2), but also increases the dimension of the aforementioned vector. The latter has the additional effect of alleviating the overall error, by increasing the number of observations in the least squares method, thus preventing the localization error from increasing in a smooth linear manner as was the case with Figure (10) for sampling time. This again points to a trade-off this time in the optimum choice of \(N\), the increase of which, will on the one hand increase the DAL error, and on the other hand lessen the overall error by increasing the number of equations in the linear system of (6), while the extra equations will be noisier. What is most interesting is the effect of the proposed weighting on this parameter. According to the figure, with the weighting introduced in section (IV) one can annul the effect of increasing \(N\) on the Data Acquisition Latency, and thus freely exploit the former to cancel the measurement noises.

This will in turn enhance the robustness of the proposed scheme against distance measurement errors.

VI. CONCLUSION

It was shown that the time-difference between the data acquisition and localization which is intrinsic to mobile WSN, can prove to be an impediment to the operation of the traditional distance estimation and position computation techniques. Accordingly, a new kind of error called Data Acquisition Latency error, was introduced and its properties were investigated. Moreover, a weighted linear system of equations was suggested that can help mitigate the effect of DAL error on the lateration process. The simulation results confirmed the superior performance of the proposed method.

REFERENCES