

Elastic Response Spectra

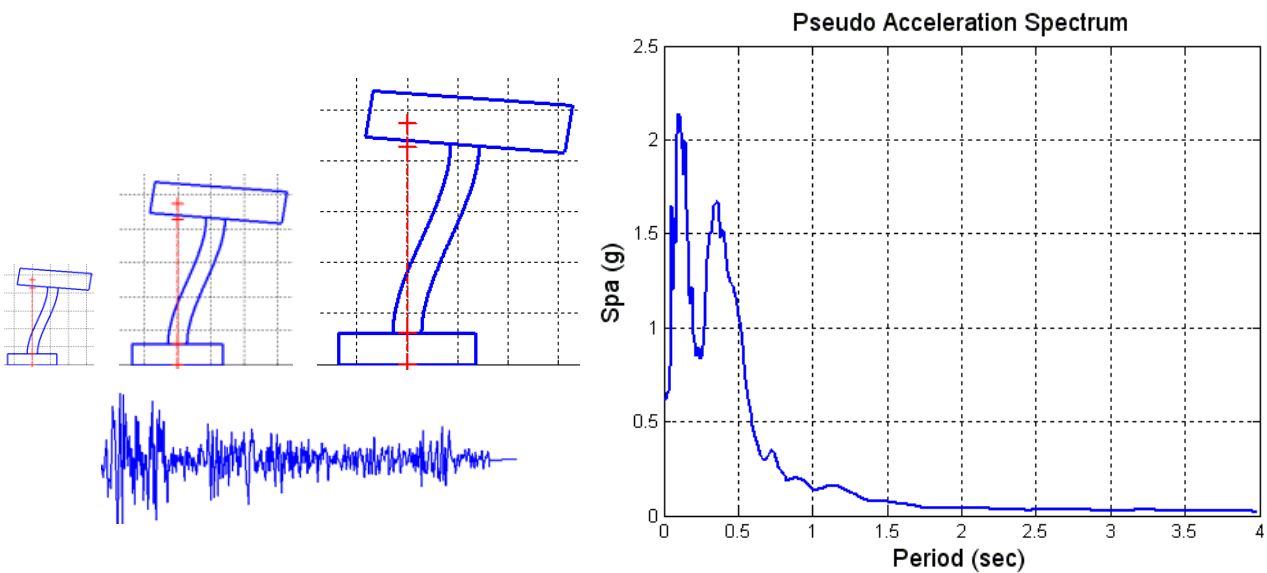
By Newmark's β Method

Carleton University

© Mostafa Tazarv

Graduate Student

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Introduction

To design a structure against earthquake, one of the most recommended methods by design specifications is “response spectrum analysis” in which rather than time history analysis, maximum responses are estimated by this method. To do so, first, natural frequencies (or period) should be obtained by means of modal analysis of desired structure. Then, we need a curve which covers all the possible frequencies versus maximum responses called “response spectrum”. Specifically, this curve represents the maximum responses of several structures (each frequency represents a structure) under an earthquake. Response can be displacement, velocity or acceleration. However, for response spectrum analysis, displacement spectrum is of interest. Finally, for each mode maximum values can be estimated by this curve and by an appropriate combination rule such as SRSS or CQC, maximum responses of the structure can be obtained.

The main goal of the posted function (**SPEC**) is to generate elastic response spectra including displacement, pseudo velocity and pseudo acceleration spectra needed in the response spectrum analysis of structures. Under an earthquake, for desired range of period (or frequency) Eq. 1 will be numerically solved by Newmark linear method. Maximum displacement will be captured for each period (T) and finally, plot of these maximum displacements versus period will be “displacement spectrum (S_d)”. Based on this spectrum, pseudo velocity spectrum (S_{pv}) and pseudo acceleration spectrum (S_{pa}) can be obtained by Eq. 2 and Eq. 3.

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2u = -\ddot{u}_g \quad (1)$$

where

ω ($= \frac{2\pi}{T}$) is natural frequency,

ξ is damping ratio and

\ddot{u}_g is ground motion acceleration of the earthquake.

$$S_{pv} = \omega S_d \quad (2)$$

$$S_{pa} = \omega^2 S_d \quad (3)$$

Function Help: SPEC

This is a self-explanatory function indeed. However, there is an option to control end period of spectra called “endp”. For example, you may need the spectra up to 4 second then please set this factor to 4.

```
[T, Spa, Spv, Sd]=SPEC(dt, Ag, zet, g, endp)
```

% INPUTS:

% dt: Time Interval (Sampling Time) of earthquake acceleration ground motion
% Ag: Ground Motion Acceleration in g (a vector)
% zet: Damping Ratio in percent (%); e.g. 5
% g: Gravitational Constant; e.g. 9.81 m/s/s
% endp: End Period of Spectra; e.g. 4 sec

% OUTPUTS:

% T: Period of Structures (sec) (a vector)
% Spa: Elastic Pseudo Acceleration Spectrum (a vector) in g
% Spv: Elastic Pseudo Velocity Spectrum (a vector)
% Sd: Elastic Displacement Spectrum (a vector)

NOTE:

- If you don't like to have the plot of spectra, please delete lines 62-74.
- If g is in m/s^2 , then S_{pv} will be in the unit of m/s and S_d will be in the unit of m .

Example 1: Response Spectra of Elcentro Earthquake

It is desired to obtain the elastic response spectra of N-S component of 1940 Elcentro earthquake (Fig. 1) with 2% damping ratio up to 5sec. This record is also attached to the submitted files. First column is time and second column is acceleration in g. dt is 0.02sec. Please call the function as follows:

Hint to load this record in workspace: `load elcentro.dat`

Hint: `g` is 9810 mm/s²

```
[T, Spa, Spv, Sd]=SPEC(0.02, elcentro(:, 2), 2, 9810, 5)
```

Fig. 2 and 3 show the spectra. Since g is in mm/s², S_d will be in *mm* as well.

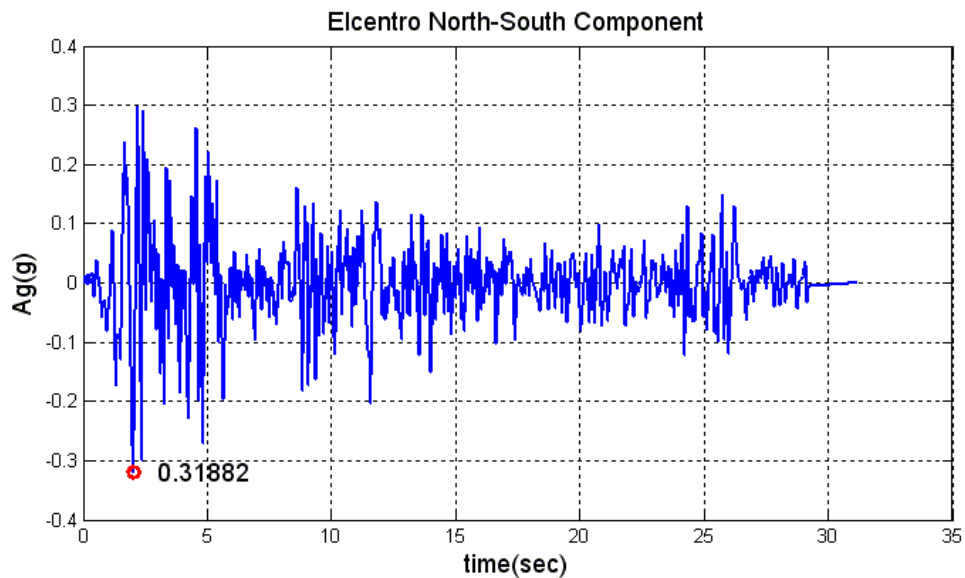


Figure 1- Ground Motion Acceleration of N-S Component of Elcentro Earthquake in g

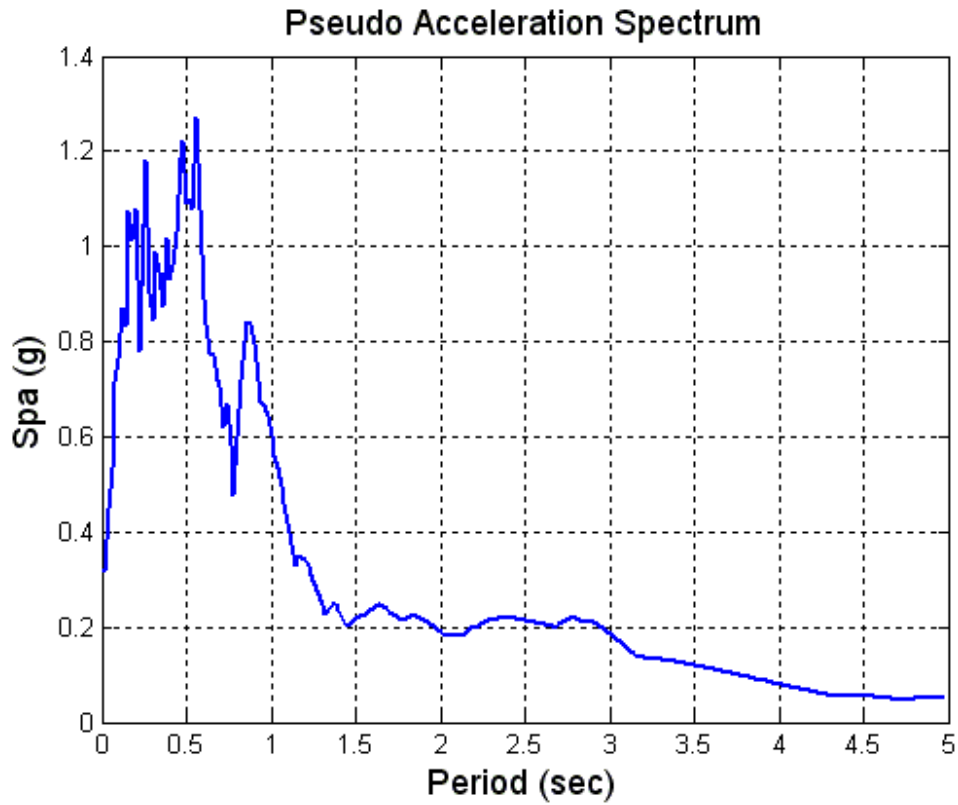


Figure 2-Pseudo Acceleration Spectrum of N-S Component of Elcentro Earthquake with $\xi = 2\%$

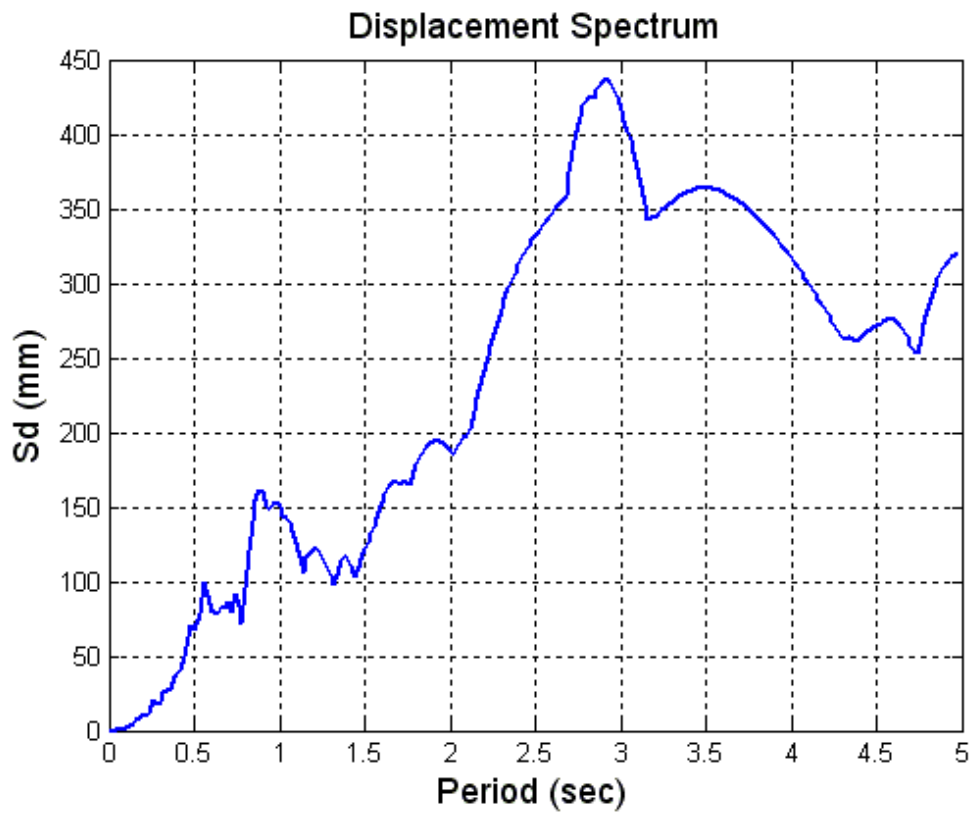


Figure 3- Displacement Spectrum of N-S Component of Elcentro Earthquake with $\xi = 2\%$

Example 2: Verification of SEPC

In this section, I will compare the spectra generated by SPEC with well-known software “SeismoSignal”. Selected earthquake is again 1940 Elcentro N-S component. However, damping ratio is 5% and end period was set to 4 sec. Fig. 4&5 compare the acceleration spectrum and displacement spectrum by these to programs.

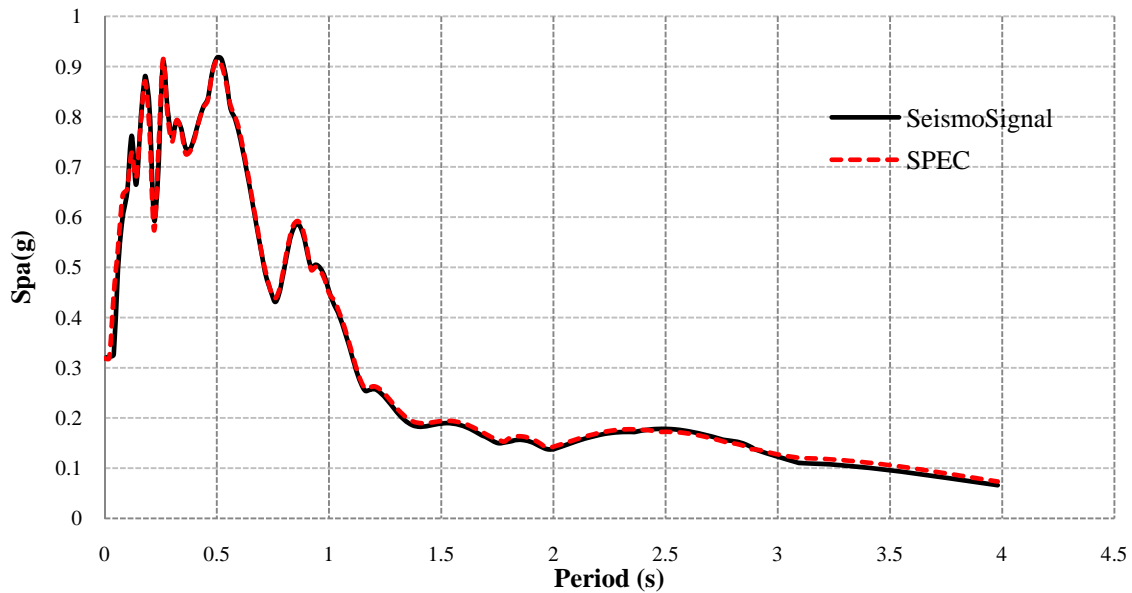


Figure 4-Comparison of Pseudo Acceleration Spectrum of N-S Component of Elcentro Earthquake with $\xi = 5\%$

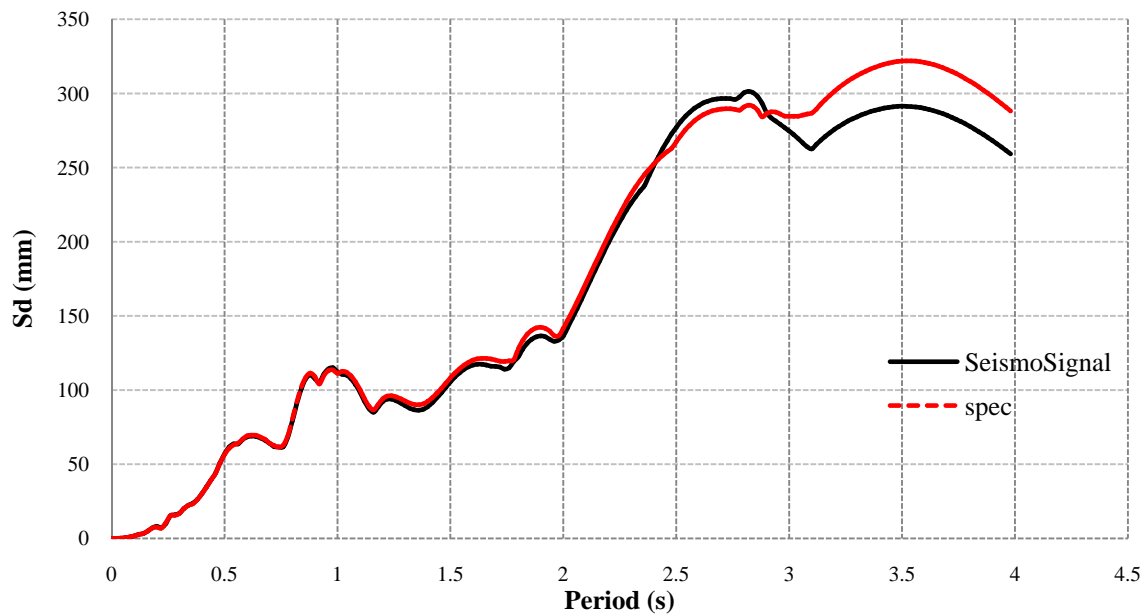


Figure 5- Comparison of Displacement Spectrum of N-S Component of Elcentro Earthquake with $\xi = 5\%$

Example 3: Verification of SEPC

Another comparison has been made. However, selected earthquake is one of the recorded ground motions of March 11, 2011 destructive earthquake in Japan (M9.0). Fig. 6 shows the site location and name (IWT006). You can download the uncorrected record form <http://www.k-net.bosai.go.jp/>. Damping ratio is 5% and end period was set to 10sec. Fig. 7&8 compare the acceleration spectrum and displacement spectrum of corrected record by these two programs.

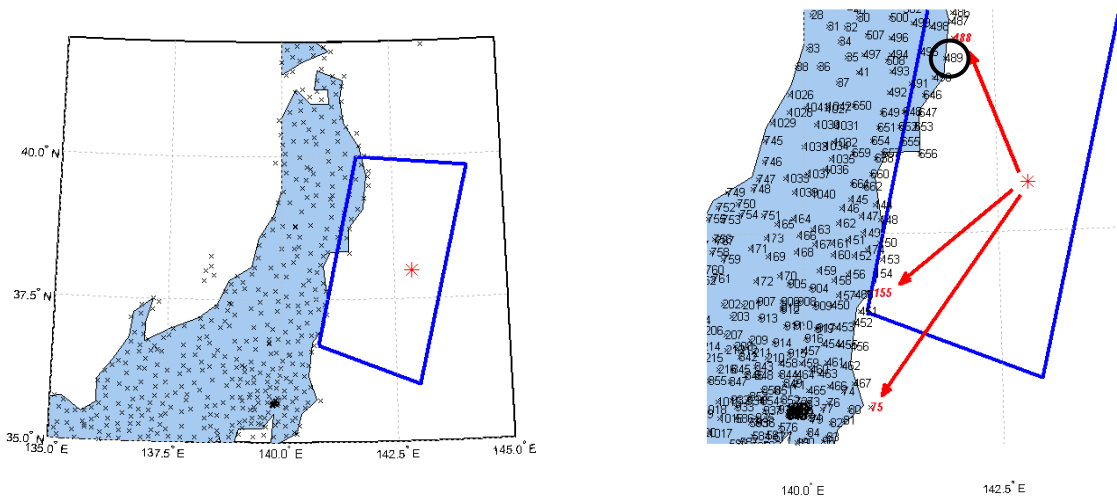


Figure 6-a) Japan Earthquake Network b) selected ground motion is IWT007

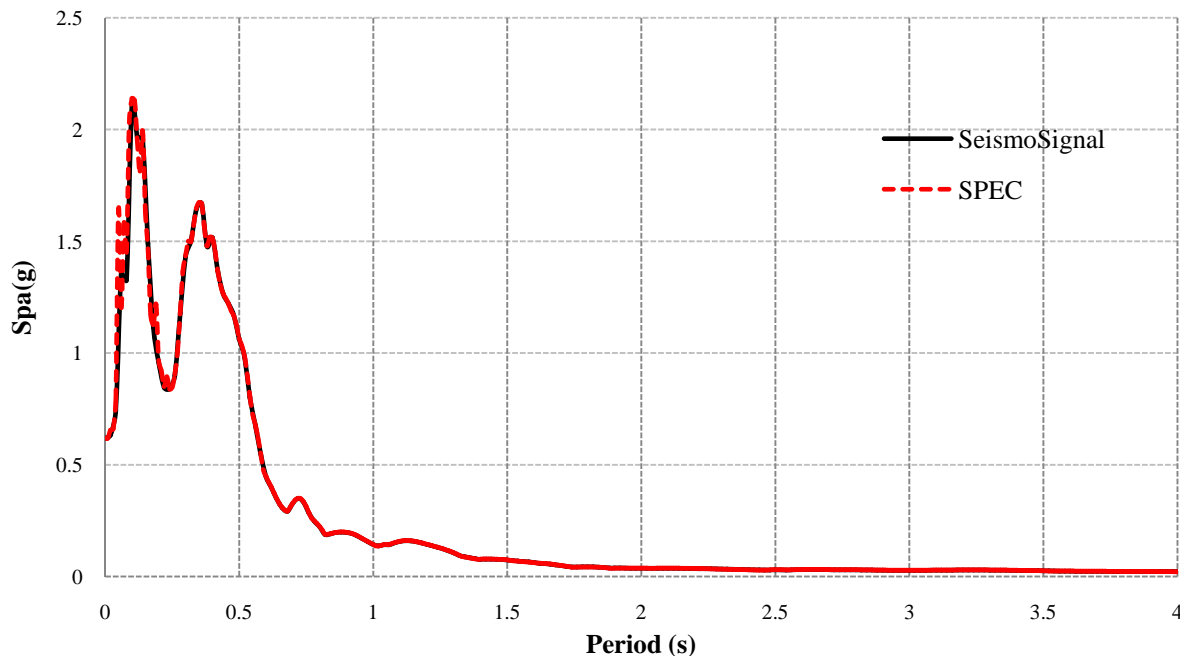


Figure 7-Comparison of Pseudo Acceleration Spectrum of N-S IWT006 of March 2011 Japan Earthquake with $\xi = 5\%$ (limited to 4sec)

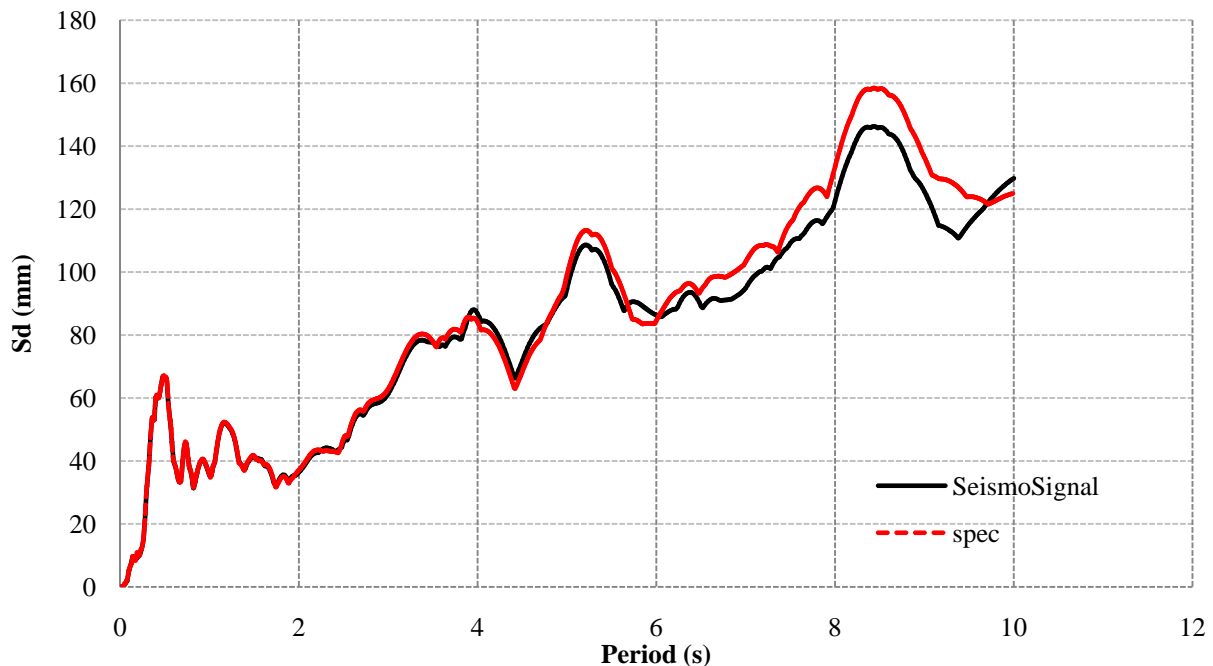


Figure 8- Comparison of Displacement Spectrum of N-S IWT006 of March 2011 Japan Earthquake with $\xi = 5\%$

I hope this function helps you in your future study, research and design project. If you are interested, I have posted another function which can carry out time history analysis of MDOF structures by mode superposition method under either earthquake or time-variant loading on DOFs. You can find it in MATLAB central called “Modal Time History Analysis of Structures” (NM.m) available online at (MATLAB may change the address):

<http://www.mathworks.com/matlabcentral/fileexchange/30866-modal-time-history-analysis-of-structures>

Reference

1. Humar J. L., “Dynamic of Structures”, *Prentice Hall*, 1990
2. Chopra A. K., “Dynamic of Structures, Theory and Application to Earthquake Engineering”, *Prentice Hall*, 1995
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5. National Research Institute for Earth Science and Disaster Prevention of Japan, <http://www.k-net.bosai.go.jp/>